

# Fundamentals of Thermodynamics Applied to Thermal Power Plants

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**Abstract** In this chapter it is reviewed the fundamental principles of Thermodynamics aiming at its application to power plants cycle analysis. The three most common thermodynamic cycles are studied starting with the Brayton cycle, the Diesel Cycle, and the Rankine cycle. These ideal cycles are thermodynamic operating models for gas turbines, diesel engines, and steam turbines, respectively. Thermal efficiencies, operating conditions and cycle variations are also analyzed. The last issue studied is the combined Brayton-Rankine cycle, which is a trend in industry due to its higher overall efficiency.

## 1 Thermodynamics Principles

In this section is presented a review of fundamental thermodynamic principles, thermodynamic properties, and the governing laws applied to processes commonly presented in thermal machines.

### *1.1 Thermodynamic Properties, Equations and Tables*

*Specific internal energy*,  $u$ —is the energy stored in the substance due to molecular motion as well as intermolecular forces. The SI unit is kJ/kg.

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*Specific enthalpy,  $h$* —is the sum of the specific internal energy and the product of pressure  $P$  versus specific volume,  $v$ . The SI unit is kJ/kg.

$$h = u + Pv \quad (1)$$

*Kinetic energy,  $KE$* —is the energy a system of mass  $m$  possesses due to the macro fluid motion at velocity  $V$ .

$$KE = mV^2/2 \quad (2)$$

*Potential energy,  $PE$* —is the energy due to the gravitational field  $g$  that a mass  $m$  possess in relation to a height  $z$  from a reference plane.

$$PE = mgz \quad (3)$$

*Shaft work,  $W$* —is the mechanical work produced or absorbed by the rotating shaft of the thermal machine.

*Shaft power,  $\dot{W}$* —is the mechanical power produced or absorbed by the rotating shaft of the thermal machine.

*Heat,  $Q$* —is the form of energy transferred to or from the machine due to a difference of temperatures between the machine and the surroundings, the higher temperature to the lower one.

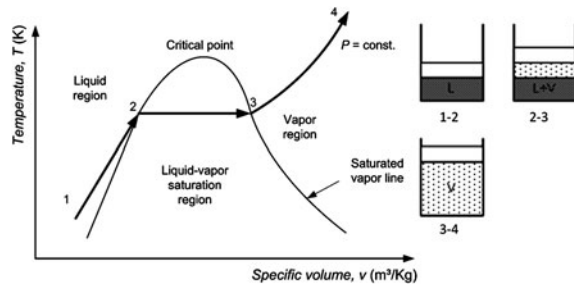
*Thermal power,  $\dot{Q}$* —is the form of energy rate transferred to or from the machine due to a difference of temperatures between the machine and the surroundings, the higher temperature to the lower one.

**Phase change:** pure substances have molecular arrangement in phases. A solid phase is the one in which the molecules do not move freely, such as in ice. In liquid phase, the molecules move partially free, such as in liquid water. Finally, in vapor phase the molecules move freely, such as in steam. All pure substances have those three phases. It is also possible to have different solid phases.

Figure 1 shows a phase diagram for water in the temperature  $\times$  specific volume plane for the liquid–vapor phases. The “bell shape” curve is more appropriately known as the saturation curve. The liquid phase is on the left and the vapor phase is on the right region. Inside the “bell shape” is the two-phase region, where liquid and vapor phases coexist in thermodynamic equilibrium. The left line is known as saturated liquid and the right one is the saturated vapor. The saturation lines meet at the critical point. All states to the left of the saturation liquid line is compressed liquid and the states to the right of the saturation vapor line are superheated vapor.

Substances change states. Consider compressed liquid water at, say, room temperature and normal pressure indicated by state 1 in the piston-cylinder setup on the right of Fig. 1. As heat is supplied at constant pressure, the system temperature increases until the liquid saturation line is achieved at state 2. If heat continues to be supplied a liquid–vapor phase change takes place and vapor bubbles arise until all the liquid phase undergoes a vaporization process and only vapor is seen inside the piston-cylinder device at state 3, or saturated vapor. On continuing supplying heat the saturated vapor becomes superheated vapor, state 4.

**Fig. 1** Liquid-vapor saturation curve in the temperature-specific volume plane and an illustration of a liquid-vapor phase change process at constant pressure



Of course, if one starts as a superheated vapor (state 4) a liquid state 1 can also be attained by removing heat from the system. If the experiment is carried out at a higher pressure, the same behavior will be observed, except that the phase change will start at a higher temperature.

There is a direct correspondence between pressure and temperature during a phase change process, which is known as the saturation curve. For each substance, including water, there is a specific temperature where a phase change will occur at a given pressure. Conversely, there is a specific pressure where a phase change will occur at a given temperature. However, for pressure above the critical pressure, there will be no phase change, as the two saturation lines meet as at the critical point as seen in Fig. 1. Therefore, above the critical pressure and temperature there will be no liquid-vapor phase change.

The process illustrated in Fig. 1 takes place at a constant pressure, known as isobaric, which is imposed on the system by the piston weight plus local atmospheric pressure. Other relevant thermodynamic processes are: (a) isothermal—constant temperature; (b) isochoric—constant specific volume; (c) adiabatic—no heat transfer to or from the system; (d) reversible process—no “losses” in the process. Of course, these processes are general and they can occur with or without any phase change.

Precise thermodynamic properties of water and many other substances can be found in tables presented in basic thermodynamic books. Normally, there are two sets of tables for water. One is valid only for the liquid-vapor saturation region, and the other for the superheated vapor region. The saturation table provides saturation liquid and vapor properties, while the other table provides superheated vapor properties.

*Vapor quality,  $x$* —is defined as the ratio between the vapor mass,  $m_v$ , and the total mass,  $m_T$ , in a given system. Vapor quality is a thermodynamic property valid only for the two-phase region or saturation region, where a mixture of liquid and vapor are at thermodynamic equilibrium.

$$x = \frac{m_v}{m_T} \quad (4)$$

Thermodynamic properties such as specific volume, specific internal energy, and specific enthalpy are averaged by the vapor quality in the two-phase region from the saturated liquid (subscript “L”) and vapor (subscript “V”) corresponding

values. Average saturation properties can be obtained from a saturation table such as the one for water.

$$v = xv_v + (1 - x)v_L \quad (5)$$

$$u = xu_v + (1 - x)u_L \quad (6)$$

$$h = xh_v + (1 - x)h_L \quad (7)$$

**Equations of State and Specific Heats** thermodynamic properties are related to each other by equations of state. Most equations of state relate pressure, specific volume, and temperature, and have the general form given by  $f(P, v, T) = 0$ . An equation of state, or simply, EOS can be a very complex mathematical function having several coefficients and constants and can be valid for both liquid and vapor regions. Also, equations of state can be presented in graphical form and tables. Saturation and superheated tables are good examples of precise equations of state. However, all equations of state valid for the vapor phase do have a low pressure limit given by the ideal equation of state given by

$$Pv = RT \quad (8)$$

where the temperature must be in absolute value, and  $R$  is the particular gas constant, which is given by the ratio between the universal ideal gas constant,  $\mathfrak{R}$ , and the gas molecular weight,  $M$ .

$$R = \frac{\mathfrak{R}}{M} \quad (9)$$

Some values of  $\mathfrak{R}$  are  $8.314 \text{ kJ/kgmol.K} = 1.987 \text{ kcal/kgmol.K} = 847.7 \text{ kgf/kgmol.K}$ .

All vapors and gases agree with the ideal EOS for pressures much lower than the critical pressure and the ideal EOS can be used for system pressure lower than 5% for engineering purposes. The lower the pressure, the better the agreement. Also, if the system temperature is about twice the critical temperature, the ideal behavior is valid as well.

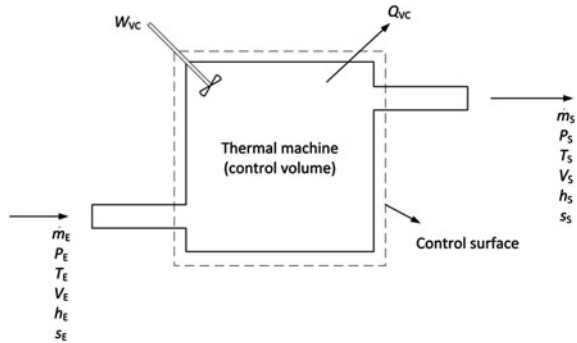
*Specific heat at constant pressure*,  $C_p$  strictly speaking this thermodynamic property is defined in terms of partial derivative. However, when the substance is an ideal gas, it can be defined as

$$C_p = \frac{dh}{dT} \quad (10)$$

*Specific heat at constant volume*,  $C_v$  strictly speaking this thermodynamic property is defined in terms of partial derivative. However, when the substance is an ideal gas, it can be defined as

$$C_v = \frac{du}{dT} \quad (11)$$

**Fig. 2** Control volume for energy balance analysis



The first specific heat involves specific enthalpy, and the other one, specific internal energy. For liquids and solids, both specific heats are very close to each other and one can say simply *specific heat*,  $C$ .

For an ideal gas, there is a very useful relationship between these two specific heats given by

$$C_p - C_V = R \quad (12)$$

The ratio between the two specific heats is quite important in analyzing thermodynamic process. For this it is defined the specific heat ratio,  $\gamma$ , as:

$$\gamma = \frac{C_p}{C_V} \quad (13)$$

The above property is very useful on studying some processes in ideal gas.  $\gamma$  is bound by two limits:  $1 \leq \gamma \leq 5/3$ . For air  $\gamma = 1.4$ .

## 1.2 First Law of Thermodynamics Analysis for Control Volumes

Thermal machines convert chemical energy in shaft work by burning fuel (heat) in a combustion chamber. In doing so, mass fluxes of air and fuel enter the machine and combustion products exit it. In a working machine, energy in its several forms is presented in the conversion process, such as heat, shaft work, enthalpy, and chemical energy. Even though energy is transformed from one form into another, the overall amount of energy must be conserved as stated by the First Law of Thermodynamics or Law of Conservation of Energy. In order to establish the First Law consider the schematics in Fig. 2 showing a control volume around a thermal

machine. All relevant forms of energy and variables fluxes are shown along with shaft power and heat flux.

Energy balance for the control volume in Fig. 2 results in

$$\left(\frac{dE}{dt}\right)_{CV} = \sum \dot{m}_i \left(h_i + \frac{V_i^2}{2} + Z_i\right) - \sum \dot{m}_o \left(h_o + \frac{V_o^2}{2} + Z_o\right) + \dot{Q} - \dot{W}. \quad (14)$$

The total energy  $E$  is the instantaneous total energy within the control volume (such as a thermal machine). The first two terms in the r.h.s. are the specific enthalpy and the kinetic and potential specific energies associated with all inlet mass fluxes,  $\dot{m}_i$ , and outlet mass fluxes,  $\dot{m}_o$ .  $\dot{Q}$  is the rate of heat the control volume exchange with the environment, and  $\dot{W}$  is the shaft power generated by the control volume. The units of shaft power and rate of heat transfer is kW in the SI system. Positive values are for heat gained and for net work produced.

Most machines operate in steady state. In steady states, the heat rate and shaft power along with the inlet and the outlet conditions and thermodynamic properties do not change and, consequently, the total energy do not vary in time. Therefore, the time rate is null and the Eq. 14 can be simplified to obtain

$$\sum \dot{m}_i \left(h_i + \frac{V_i^2}{2} + Z_i\right) + \dot{Q} = \sum \dot{m}_o \left(h_o + \frac{V_o^2}{2} + Z_o\right) + \dot{W}. \quad (15)$$

A particular case arises when there is only one inlet and one outlet mass flux. So,

$$h_i + \frac{V_i^2}{2} + Z_i + q = h_o + \frac{V_o^2}{2} + Z_o + w. \quad (16)$$

where lower cases are used for heat and work terms on mass basis, i.e., heat per unit of mass and work by unit of mass. Also, inlet and outlet mass fluxes are equal.

### ***1.3 Second Law of Thermodynamics Analysis for Control Volumes***

The rate of entropy generated in a control volume (Fig. 2) can be written according to Eq. 17

$$\left(\frac{dS}{dt}\right)_{CV} \geq \sum \dot{m}_i s_i - \sum \dot{m}_o s_o + \sum \frac{\dot{Q}_{CV}}{T}. \quad (17)$$

where,  $S$  is the total instantaneous entropy of the control volume,  $s_i$  and  $s_o$  are the specific entropy associated with the inlet and outlet mass fluxes,  $T$  is the control volume surface temperature where heat is exchanged with the surrounding environment at a given rate,  $\dot{Q}_{cv}$ . The inequality is to take into account the irreversibilities that can occur. By adding a positive term to consider the time rate at which

irreversibility occurs,  $\dot{S}_{gen}$ , one can drop the inequality. Also, in steady state conditions, the control volume instantaneous entropy remains constant. Therefore, with these two assumptions, one can obtain:

$$\sum \dot{m}_o s_o - \sum \dot{m}_i s_i = \sum \frac{\dot{Q}_{CV}}{T} + \dot{S}_{gen} \quad (18a)$$

If there is only one inlet and one outlet, then

$$\dot{m}(s_o - s_i) = \sum \frac{\dot{Q}_{CV}}{T} + \dot{S}_{gen} \quad (18b)$$

For an adiabatic process, there is no heat transfer, therefore

$$\dot{m}(s_o - s_i) = \dot{S}_{gen} \quad (19a)$$

or, considering that the entropy generation time rate is always positive, then

$$s_o \geq s_i \quad (19b)$$

where, the equality is valid for an adiabatic and reversible ( $\dot{S}_{gen} = 0$ ) process.

#### ***1.4 Reversible Work, Polytropic Process and Entropy Variation in Ideal Gases***

**Reversible Work** is the shaft work an ideal machine, such as pumps, compressors, turbines, produces or demands on carrying out a given thermodynamic process. There is a differential fundamental thermodynamic relationship derived from the combination of First and Second Laws of Thermodynamics known as the Gibbs equation, given by:

$$du = Tds - Pdv \quad (20a)$$

By differentiating the specific enthalpy (Eq. 1) and introducing the differential form into the above equation, one obtains.

$$dh = Tds + vdP \quad (20b)$$

For a reversible process, the differential form of the specific entropy is

$$ds = \frac{\delta q}{T} \quad (21)$$

On the other hand, the First Law in differential form neglecting the kinetic and potential energy variations can be obtained from Eq. 16, this is

$$\delta q = dh - \delta w \quad (22)$$

Finally, substituting Eqs. 21 and 22 into Eq. 20b after integration, one obtains.

$$w = - \int_{P_i}^{P_o} v dP \quad (23)$$

Equation 23 is a remarkable expression that allows one to calculate the reversible work by unit of mass for any reversible thermodynamic process in steady state in a control volume. The minus signal comes from the convention of positive work produced by the control volume.

**Polytropic Process** Thermal machines such as internal combustion engines and gas turbines are modeled by air standard cycles, such as Brayton and Diesel cycles discussed later in this chapter. In the modeling process of those thermodynamic cycles, an amount of air undergoes several thermodynamic processes which can be analyzed by using the ideal gas behavior. In doing so, simple working equations arise. Therefore, it is important to analyze the several thermodynamic process associated with an ideal gas transformations. In a broad sense, many useful thermodynamic reversible processes can be analyzed at once by using the concept of polytropic process. Those processes include isothermal, isentropic, isobaric, isochoric, a general process with or without heat transfer as it will be seen. Air standard cycles can also be used to analyze other devices, such as the Ranque-Hilsh or vortex tube, as presented by Simões-Moreira [1].

A general polytropic process is the one that obeys the following relationship between pressure and specific volume

$$Pv^n = \text{const} \quad (24)$$

where  $n$  is the polytropic coefficient. It can assume any value. Some particular values of  $n$  represent a special thermodynamic process, such as:

- Isobaric process ( $p = \text{constant}$ ):  $n = 0$ ;
- Isothermal process ( $T = \text{constant}$ ):  $n = 1$ ;
- Isentropic or adiabatic reversible process ( $s = \text{constant}$ ):  $n = \gamma$ ;
- Isochoric process ( $v = \text{constant}$ ):  $n = \infty$

The reversible work by unit of mass can now be calculated for an ideal gas from its definition (Eq. 23) by a process varying from  $P_1$  to  $P_2$

$$w = - \int_{P_1}^{P_2} v dP = -\text{const} \int_{P_1}^{P_2} \frac{dP}{P^{1/n}} \quad (25)$$

The constant can be related either to the initial state 1 or the final one 2, as needed or desired, i.e.,  $P_1 v_1^n = P_2 v_2^n = \text{const}$ . In order to carry out the integration, it is necessary to separated the integral in two situations, one is for  $n = 1$  and the other for  $n \neq 1$ .

If  $n = 1$  (isothermal process), then from Eq. 25, one obtains:



$$w = -const \int_{P_1}^{P_2} \frac{dP}{P} = -P_1 v_1 \ln\left(\frac{P_2}{P_1}\right) = -P_2 v_2 \ln\left(\frac{P_2}{P_1}\right) = -RT \ln\left(\frac{P_2}{P_1}\right) \quad (26)$$

If  $n \neq 1$  (any other polytropic process), then from Eq. 25, one obtains:

$$w = -const \int_{P_1}^{P_2} \frac{dP}{P^{1/n}} = -\frac{n}{n-1} (P_2 v_2 - P_1 v_1) = -\frac{nR}{n-1} (T_2 - T_1) \quad (27)$$

**Entropy Variation in Ideal Gases** working equations can be obtained for entropy variation in ideal gases. In order to obtain those equations, let us start off with the differential form of the fundamental thermodynamic relationship given by Eq. 20 and substitute both  $du = C_v dT$  from Eq. 11 and the ideal gas equation of state from Eq. 8 into it, to obtain

$$ds = C_v \frac{dT}{T} + R \frac{dv}{v} \quad (28)$$

Next, by integrating it between two states of interest, it yields,

$$\Delta s = s_2 - s_1 = C_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right) \quad (29)$$

Alternatives forms of the above equations can also be obtained depending on the selecting the two other independent variables out pressure, temperature and specific volume set. They are:

$$\Delta s = s_2 - s_1 = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right) \quad (30)$$

and

$$\Delta s = s_2 - s_1 = C_v \ln\left(\frac{P_2}{P_1}\right) + C_p \ln\left(\frac{v_2}{v_1}\right) \quad (31)$$

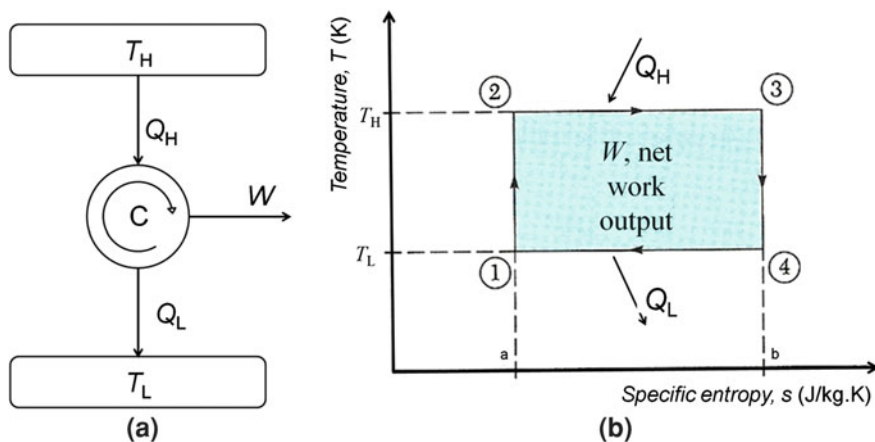
Isentropic process occurs in analysis of ideal machines as we will see in next sections. By equating Eqs. 29–30 to zero, it is possible to obtain corresponding  $P$ - $v$ - $T$  isentropic relationships. They are in order:

$$T_1 v_1^{\gamma-1} = T_2 v_2^{\gamma-1}, \quad (32)$$

$$P_1 v_1^\gamma = P_2 v_2^\gamma, \quad (33)$$

and

$$P_1^{\frac{1-\gamma}{\gamma}} T_1 = P_2^{\frac{1-\gamma}{\gamma}} T_2, \quad (34)$$



**Fig. 3** **a** Schematics of a heat engine; **b**  $T$ - $s$  diagram for a Carnot cycle

### 1.5 The Carnot Cycle

On studying heat engines and thermal machines, one is faced with a question very relevant: Given two sources of thermal energy at two different temperatures, one at a high temperature  $T_H$  and the other at a low temperature  $T_L$ , what is the maximum conversion of heat drawn from the source at high temperature that can be converted into useful work in an ideal heat engine (reversible one) that operates continuously in a closed thermodynamic cycle? First, the Kelvin-Planck statement of the Second Law of Thermodynamics tells us that it is impossible to have a heat engine that will convert all the heat received from the high temperature source,  $Q_H$ , into useful work in a thermodynamic cycle. It is necessary to reject part of the received heat to the low temperature source,  $Q_L$ . In other words: it is impossible to have a 100% efficiency heat engine. A schematic of an operating heat engine according to Kelvin-Planck is shown in Fig. 3a.

Second, Carnot devised that the heat engine that can achieve the maximum efficiency in continuously converting heat into work operating between the two heat sources is the one made up of four reversible processes as illustrated in the temperature-entropy diagram in Fig. 3b, which are:

- process 1–2—temperature raise from  $T_L$  to  $T_H$  in an adiabatic reversible process (isentropic);
- process 2–3—heat addition,  $Q_H$ , in an isothermic reversible process at  $T_H$ ;
- process 3–4—temperature decrease from  $T_H$  to  $T_L$  in an adiabatic reversible process (isentropic);
- process 4–1—heat rejection,  $Q_L$ , in an isothermic reversible process at  $T_L$ .

The thermal efficiency of any power cycle,  $\eta_{th}$ , is the ratio of the network,  $W$ , and the heat received,  $Q_H$ .

$$\eta_{th} = \frac{W}{Q_H} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H} \quad (35)$$

where, the First Law has also been used, i.e.,  $W = Q_H - Q_L$ .

From the  $T$ - $S$  in diagram Fig. 3b, it is possible to notice that both heat addition and rejected are associated with entropy variation, i. e.

$$Q_H = T_H \Delta S \quad (36a)$$

and

$$Q_L = T_L \Delta S \quad (36b)$$

Therefore, substituting above equations into Eq. 35, one obtains the final form of the Carnot efficiency,  $\eta_C$ , which is:

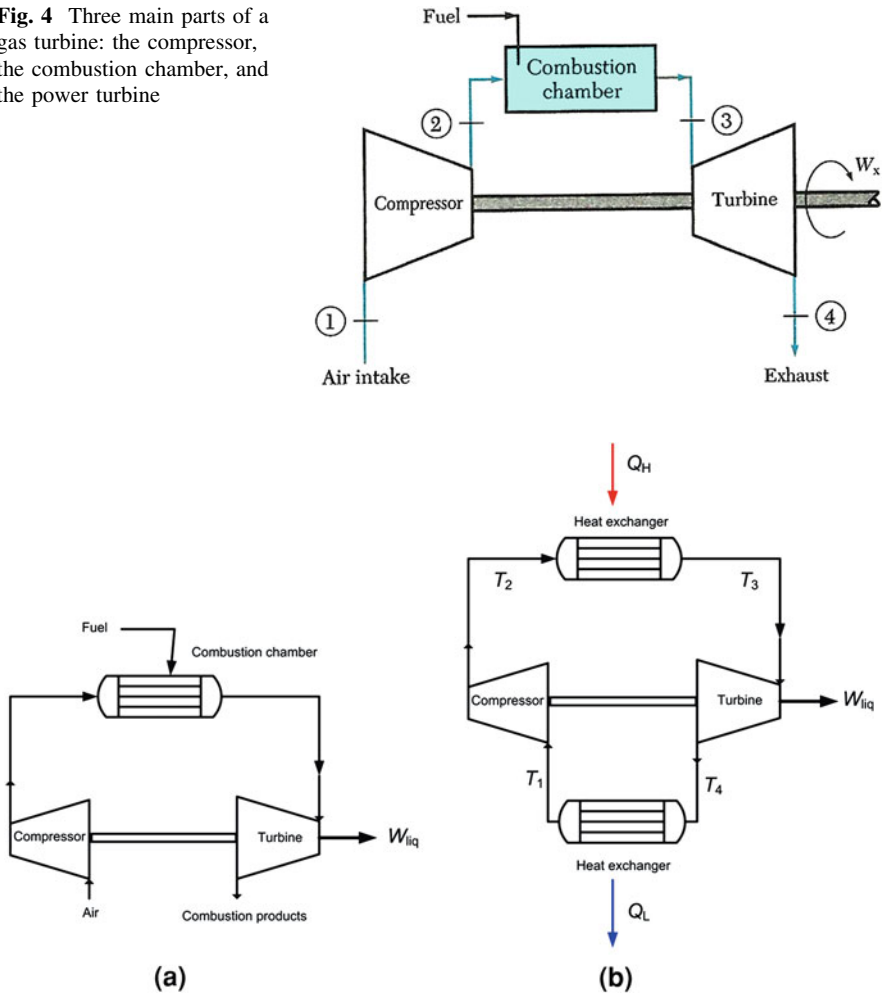
$$\eta_C = 1 - \frac{T_L}{T_H} \quad (37)$$

This remarkable result shows that the maximum conversion of heat into work in heat engine operating continuously between two heat sources is limited by the ratio between the two heat sources temperatures. The lower the temperature ratio, the higher the Carnot efficiency. As a final remark, no 100% conversion can take place because it would require either a 0 K low temperature source, or an extremely high temperature source (mathematically, an infinite one), or both.

## 2 Gas Turbine Cycles

Gas turbines are complex turbo machines made up of thousands of parts. Nevertheless, gas turbines have three main parts that perform the fundamental thermodynamic processes involved in the mechanical shaft power production from the fuel chemical energy as illustrated in Fig. 4. First, the income atmospheric air must undergo a compression process in the compressor section where both pressure and temperature are increased. Next, the compressed air is driven to a combustion chamber where fuel is injected into the compressed air stream and burnt increasing the temperature at a constant pressure process. Finally, the combustion products at a high temperature and pressure are expanded in the power turbine section generating shaft power to drive the compressor as well as an electrical generator or any other rotary device attached to the rotary shaft. The combustion products are exhausted through a nozzle into the atmosphere.

**Fig. 4** Three main parts of a gas turbine: the compressor, the combustion chamber, and the power turbine



**Fig. 5** **a** Open cycle; **b** closed air standard Brayton cycle

## 2.1 Simple Brayton Cycle

In an actual gas turbine, the working fluid changes from atmospheric air to combustion products that exhaust back to the atmosphere, as illustrated in Fig. 5a. However, in order to evaluate the machine from the thermodynamic point-of-view, some assumptions are needed. Firstly, the working fluid is assumed to be plain air, without any chemical transformation due to the combustion. In doing so, the air-fuel combustion process is replaced by a heat addition process at a constant pressure. Secondly, the exhaust and admission processes are replaced by a heat transfer process to the environment, which makes the air to flow continuously in a

closed loop as indicated in Fig. 5b. In the closed cycle, air at environment pressure and temperature is first compressed, next it receives heat  $Q_H$  and it is followed by an expansion process in the turbine section to, finally, reject heat  $Q_L$  at constant pressure. This is the Air-Standard Brayton Cycle.

Having the cycle of Fig. 5b in mind along with the ideal gas behavior and constant thermodynamic properties one may obtain the working equations from an energy balance (Eq. 16) for each cycle component:

$$\text{heat addition : } q_H = h_3 - h_2 = C_P(T_3 - T_2) \quad (38)$$

$$\text{heat rejection : } q_L = h_4 - h_1 = C_P(T_4 - T_1) \quad (39)$$

$$\text{compression work : } w_{\text{comp}} = h_2 - h_1 = C_P(T_2 - T_1) \quad (40)$$

$$\text{turbine work : } w_{\text{turb}} = h_3 - h_4 = C_P(T_3 - T_4) \quad (41)$$

$$\text{cycle net work : } w = w_{\text{turb}} - w_{\text{comp}} \quad (42)$$

Equations 38 through 42 are on mass basis whose unit is kJ/kg in the international system of units, SI. Also, both the kinetics and potential forms of energy have been neglected.

Figures 6a and 6b gives two important thermodynamic diagrams for cycle analysis. The first one is the temperature-entropy diagram and the second one is the pressure-specific volume diagram. The simple Brayton cycle formed by its four basic ideal gas processes is indicated in both diagrams. The cycle net work is given by the enclosed area shown in figures. First, air is compressed ideally (isentropic) in the compressor (process 1–2) increasing both pressure and temperature at expenses of using compression work ( $w_{\text{comp}}$ ) which is supplied by the turbine itself. Second, heat ( $q_H$ ) is added at constant pressure making up the process 2–3, which heats up the air to the highest cycle temperature,  $T_3$ . Next, the high pressure and temperature air undergoes an expansion process (process 3–4) generates work ( $w_{\text{turb}}$ ) enough to drive the compressor and produce net shaft work ( $w$ ). Finally, heat ( $q_L$ ) is rejected to the environment (process 4–1) at constant low pressure closing the cycle.

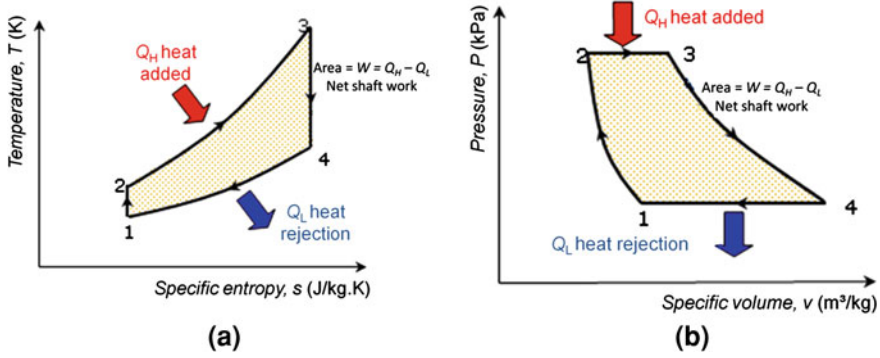
The thermal efficiency,  $\eta_{th}$ , of a cycle is defined as the ratio between the cycle net work and heat added, as given by Eq. 35. By applying the First Law for the whole cycle, one easy can show that  $w = q_H - q_L$ . Therefore, one obtains:

$$\eta_{th} = 1 - \frac{q_L}{q_H} \quad (43)$$

Finally, using Eqs. 38 and 39, it yields:

$$\eta_{th} = 1 - \frac{C_P(T_4 - T_1)}{C_P(T_3 - T_2)} = \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)} \quad (44)$$

By examining the temperature-entropy diagram in Fig. 6a, one can easily notice that  $T_3$  is the maximum cycle temperature, also known as the firing temperature, while  $T_1$  is the minimum one (usually the environment temperature).



**Fig. 6** Simple Brayton cycle in **a** temperature-specific entropy diagram; **b** pressure-specific volume diagram

By using isentropic ideal gas relationships between pressure and temperature (Eq. 34), it is straightforward to show that:

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} \text{ and } \frac{p_3}{p_4} = \left(\frac{T_3}{T_4}\right)^{\frac{\gamma}{\gamma-1}} \quad (45)$$

Also, from the diagram of Fig. 6b, one may notice that

$$r = \frac{p_2}{p_1} = \frac{p_3}{p_4} \quad (46)$$

where,  $r$  is the ratio of maximum and minimum cycle pressures. Therefore, after substituting it into Eq. 45, one may show that  $T_3/T_2 = T_4/T_1$ . Finally, substituting that in Eq. 44, one obtains:

$$\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{1}{r^{\frac{\gamma-1}{\gamma}}} \quad (47)$$

Figure 7 shows a graphics of the thermal efficiency as a function of the pressure ratio for air as given by Eq. 47 for air (value of  $\gamma = 1.4$ ).

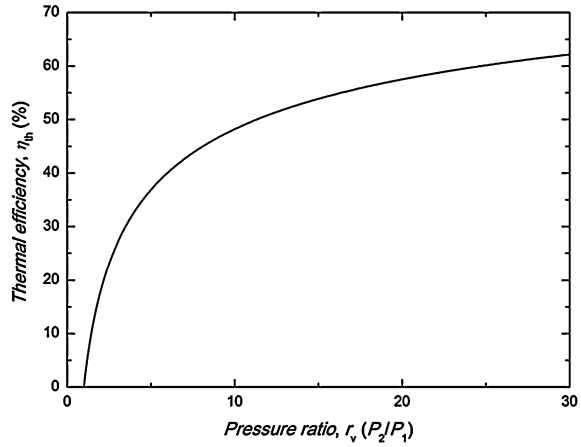
Generally, it is not enough to carry out a simple thermal efficiency analysis to find the best operational condition of a Brayton cycle. A non-negligible amount of work is required to compress the air from the inlet to the maximum cycle pressure and this work must be supplied by the turbine itself. Therefore, one should examine the net work produced by the system compressor-turbine as a whole. In order to achieve that, first subtract Eq. 41 from Eq. 40, to obtain:

$$\text{Net work : } w = w_{turb} - w_{comp} = C_p(T_3 - T_4) - C_p(T_2 - T_1) \quad (48)$$

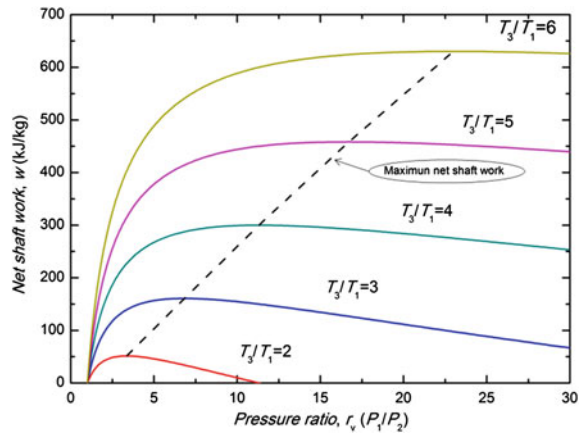
After a few manipulations using previous equations, the net shaft work is given by:

$$w = C_p T_1 \left[ \left( \frac{T_3}{T_1} \right) \times \left( 1 - \frac{1}{r^{\frac{\gamma-1}{\gamma}}} \right) - \left( r^{\frac{\gamma-1}{\gamma}} - 1 \right) \right] \quad (49)$$

**Fig. 7** Thermal efficiency the simple Brayton cycle as a function of the pressure ratio



**Fig. 8** Net shaft work for several temperature ratios,  $T_3/T_1$ , for inlet air at 300 K



Further analysis of Eq. 49 indicates that the net work is a function of the ratio between maximum and minimum temperature, pressure ratio, along with two fluid thermodynamic properties. It is difficult to get a hold on the precise net shaft work dependency on each one of those variables by a simple straight analysis of that equation, except if one examines it in a parametric graphic form, as shown in Fig. 8. That figure shows the net shaft work for several temperature ratios, i.e.,  $T_3/T_1$  assuming an inlet air temperature  $T_1 = 300$  K.

The condition of maximum net work is readily obtained by using the simple rule from Calculus, i.e.,  $\partial w / \partial r_{T_3/T_1} = 0$ . After applying the condition of maximum to Eq. 49 followed by a few manipulations, one obtains the pressure ratio where a maximum net shaft work takes place for a given temperature ratio:

$$r_{\max \text{ work}} = \left( \frac{P_2}{P_1} \right) = \left( \frac{T_3}{T_1} \right)^{\frac{\gamma}{2(\gamma-1)}} \quad (50)$$

In Fig. 8 the condition of maximum net shaft work is indicated by a dashed line.

## 2.2 Inefficiencies and Actual Brayton Cycle

The actual Brayton cycle is based on real turbo machines that deviate from ideal ones (isentropic). Substantial part of the work produced in the turbine section is drawn by the compressor, which can reach figures as high as 80% of turbine shaft work. If compressor and turbine efficiencies are not high enough, no net shaft work will be generated. Therefore, it is quite important to analyze how much process losses are introduced on the overall performance of the turbine due to machine inefficiency. First, two isentropic definitions must be introduced:

Compressor isentropic efficiency,  $\eta_c$ , is defined as the ratio of ideal or isentropic compression work,  $w_{comp-s}$ , to the actual compression work,  $w_{comp-a}$ . Figure 9a indicates the ideal and the actual compression process in the  $T$ - $s$  diagram.

$$\eta_c = \frac{w_{comp-s}}{w_{comp-a}} = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{T_{2s} - T_1}{T_2 - T_1} \quad (51)$$

Turbine isentropic efficiency,  $\eta_t$ , is defined as the ratio of the turbine actual work,  $w_{turb-a}$ , to the ideal or isentropic turbine work,  $w_{turb-s}$ . Figure 9b indicates the ideal and the actual expansion process in the  $T$ - $s$  diagram.

$$\eta_t = \frac{w_{turb-a}}{w_{turb-s}} = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{T_3 - T_4}{T_3 - T_{4s}} \quad (52)$$

In Fig. 9, one can see both processes of compression (a); both processes of expansion in the turbine section (b); and, finally, one can see the overall combination (c) of those processes.

Using the definition of isentropic compression work (Eq. 51), one can obtain the following equation for the actual compression work.

$$w_{comp-a} = \frac{C_p T_1}{\eta_c} \left( r^{\frac{\gamma-1}{\gamma}} - 1 \right) \quad (53)$$

and for the actual turbine work (Eq. 52):

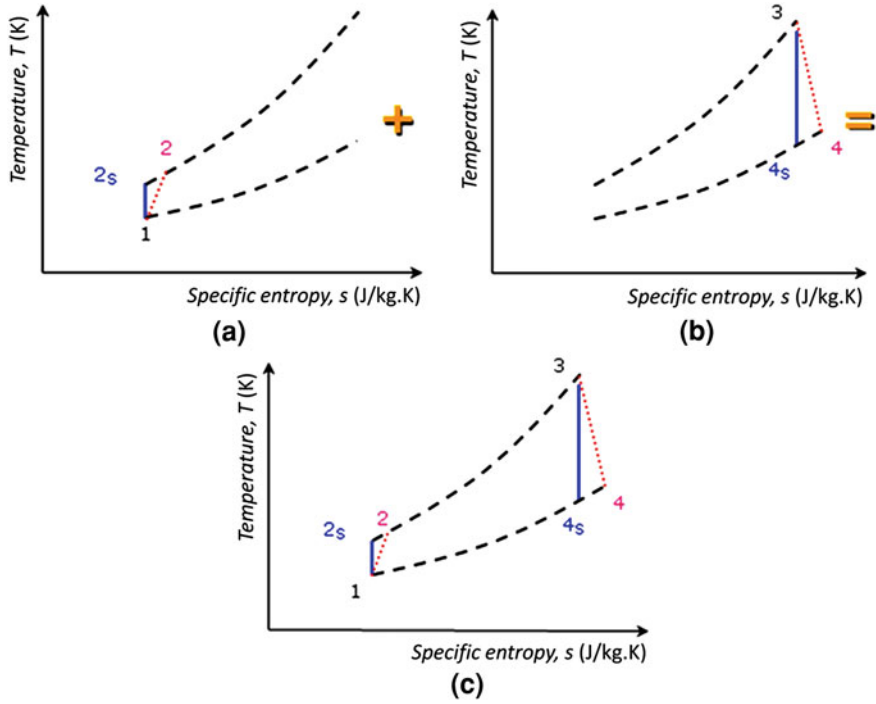
$$w_{turb-a} = \eta_t C_p T_3 \left( 1 - \frac{1}{r^{\frac{\gamma-1}{\gamma}}} \right) \quad (54)$$

By subtracting previous equations, one obtains the actual net shaft work produced by the turbine considering the losses:

$$w_a = w_{turb-a} - w_{comp-a} = C_p T_1 \left[ \eta_t \frac{T_3}{T_1} \left( 1 - \frac{1}{r^{\frac{\gamma-1}{\gamma}}} \right) - \frac{1}{\eta_c} \left( r^{\frac{\gamma-1}{\gamma}} - 1 \right) \right] \quad (55)$$

In a similar fashion, it is possible to show that the actual pressure ratio where the maximum actual net work takes place for a given temperature ratio,  $T_3/T_1$ , considering the isentropic machine efficiencies:





**Fig. 9** **a** Actual and ideal compression work; **b** actual and ideal expansion work; **c** combination of both processes

$$r_{\max \text{ work-a}} = \left( \frac{P_2}{P_1} \right) = \left( \eta_t \eta_c \frac{T_3}{T_1} \right)^{\frac{\gamma}{2(\gamma-1)}} \quad (56)$$

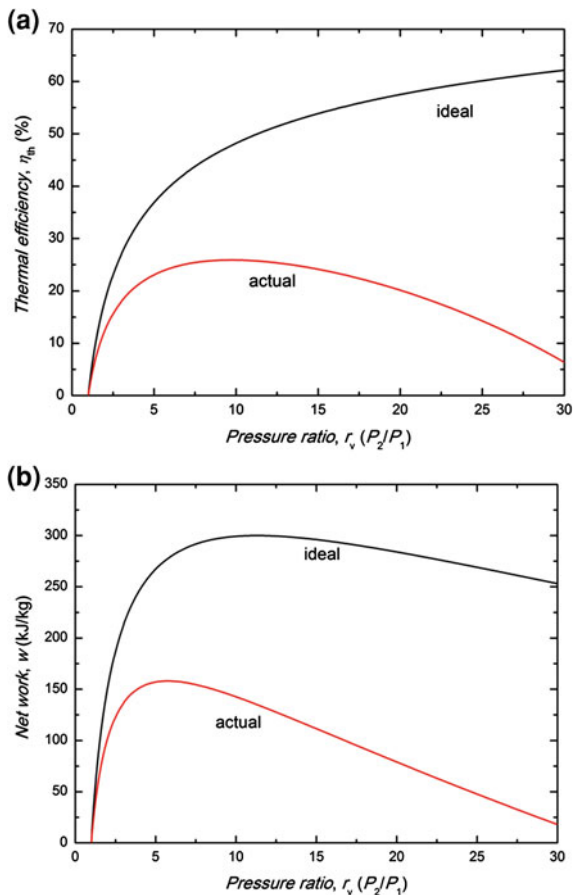
Similarly, one may obtain the actual thermal efficiency,  $\eta_{\text{th-a}}$ , as the ratio of the actual net work and the added heat.

$$\eta_{\text{th-a}} = \frac{\eta_t \frac{T_3}{T_1} \left( 1 - \frac{1}{r^{(\gamma-1)/\gamma}} \right) - \frac{1}{\eta_c} \left( r^{(\gamma-1)/\gamma} - 1 \right)}{\frac{T_3}{T_1} - \frac{1}{\eta_c} \left( r^{(\gamma-1)/\gamma} - 1 \right) - 1} \quad (57)$$

In order to verify machine efficiencies, consider a gas turbine whose compressor efficiency is 80% and turbine efficiency is 85%. Also, consider that the minimum and maximum cycle temperatures are  $T_1 = 300 \text{ K}$  and  $T_3 = 1,200 \text{ K}$ , respectively. Figure 10a displays the ideal and actual thermal efficiencies and Fig. 10b shows the net work. As seen in those graphics, machines efficiencies are quite relevant.

**Fig. 10** **a** Actual and ideal thermal efficiencies; **b** actual and ideal net work.

$T_1 = 300$  K,  $T_3 = 1,200$  K,  
 $\eta_t = 85\%$  and  $\eta_c = 80\%$



### 2.3 The Brayton Cycle With Regeneration

One striking point in Brayton cycle analysis is that the exhausting gas temperature is considerably high and often higher than the air leaving the compressor section. As heat will be added to the compressed air in the combustion chamber, a counter flow heat exchanger can be installed to pre-heat the compressed air by the exhausting combustion products, a process usually known as heat regeneration or heat recuperation. A schematic of such system is illustrated in Fig. 11. In the temperature-entropy diagram of Fig. 12,  $x$  represents the maximum compressed air temperature pre-heated prior to entering the combustion chamber. The area under states 2- $x$  represents the ideal heat, and therefore, fuel saving with heat regeneration. Also, the exhausting gas will be ideally cooled to the state  $y$  in that diagram.

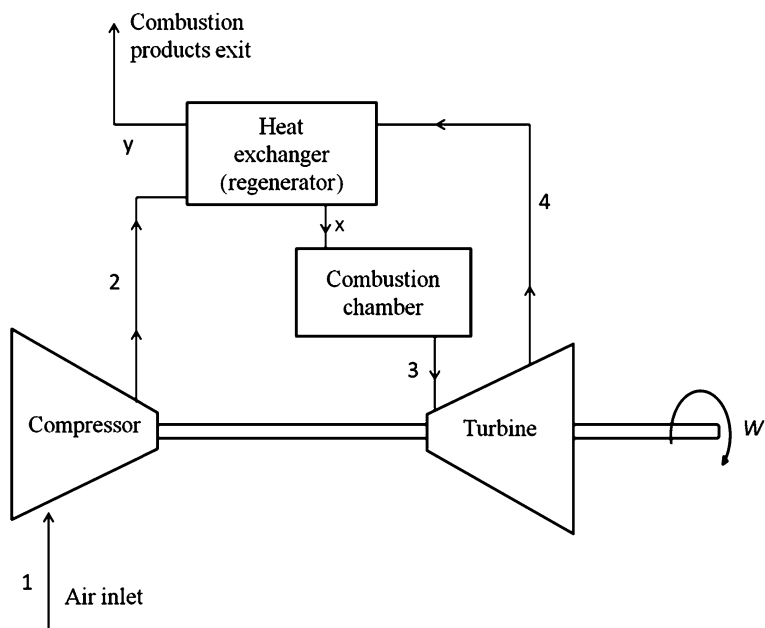
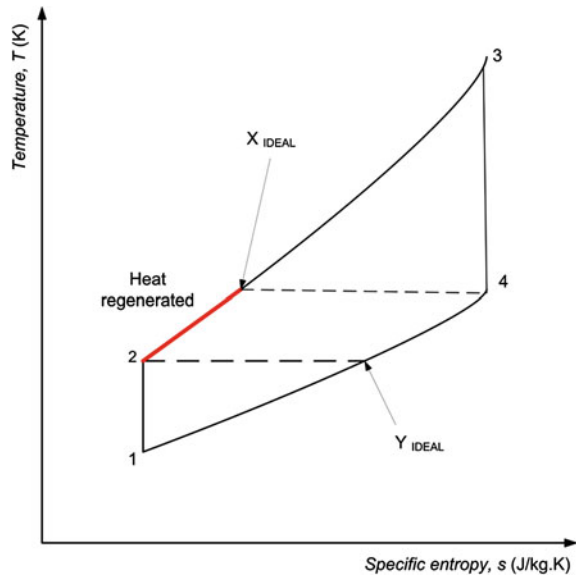
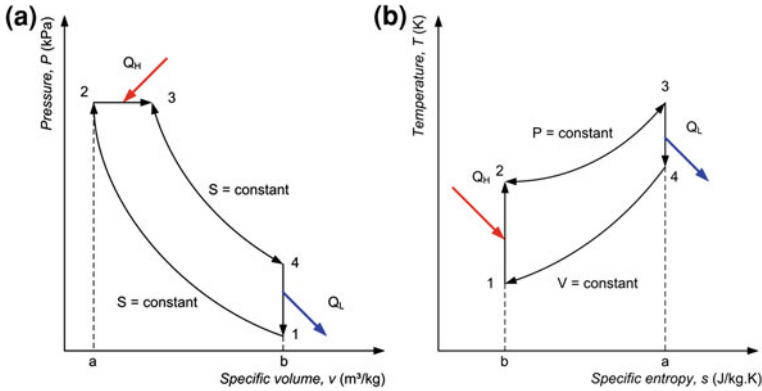


Fig. 11 Brayton cycle with heat regeneration

Fig. 12 Temperature-entropy diagram for a Brayton cycle with heat regeneration





**Fig. 13** Diesel cycle thermodynamic diagrams. **a** Pressure–volume diagram; **b** temperature–entropy diagram

Regeneration may be a good practice for open cycle gas turbine. In case of combined cycle configuration (Sect. 5) a previous study is required in order to find out whether there is an overall cycle improvement or not.

### 3 Diesel Cycle

Diesel is the air standard thermodynamic cycle used in many internal combustion engines of many small to medium thermal power plants. The working principle of an internal combustion engine is somewhat different from a closed thermodynamic cycle as it occurs also with the Brayton cycle. Working fluid composition changes from plain air to combustion products and combustion and exhaustion processes are replaced by heat transfer processes. Therefore, there is an air standard cycle that reproduces the actual machine in order to capture its main features, such as the thermal efficiency.

Figure 13 shows the two relevant diagrams for Diesel Cycle analysis. In Fig. 13a it is seen the pressure-specific volume diagram, while in Fig. 13b it can be seen the temperature-specific entropy diagram. The four ideal processes in a Diesel cycle are:

- (1) process 1–2—isentropic compression,  $w_{\text{comp}}$  in the air standard cycle. Air is compressed from pressure  $P_1$  to maximum pressure  $P_2$ . In turbocharged engines,  $P_1$  is higher than the atmospheric pressure. In naturally aspirated engines,  $P_1$  is the atmospheric pressure.
- (2) process 2–3—heat addition,  $q_H$ , at constant pressure,  $P_2 = P_3$ , takes place in the air standard cycle. In actual engine, fuel is sprayed into the compressed air as its combustion takes place generating heat.

- (3) process 3–4—in the air standard cycle compressed air at an initial high pressure and temperature  $T_3$  undergoes an isentropic expansion,  $w_{\text{exp}}$ . In the actual engine, combustion products expand from high pressure  $P_3$  to pressure  $P_4$  generating shaft power.
- (4) process 4–1—heat rejection,  $q_L$ , at constant volume,  $V_4 = V_1$ , occurs in the air standard cycle. In actual engine, the combustion products exhaust to atmosphere.

Considering the ideal processes in Fig. 13, the following energy balances can be drawn.

$$\text{heat addition : } q_H = h_3 - h_2 = C_P(T_3 - T_2) \quad (58)$$

$$\text{heat rejection : } q_L = u_4 - u_1 = C_P(T_4 - T_1) \quad (59)$$

$$\text{compression work : } w_{\text{comp}} = u_2 - u_1 = C_P(T_2 - T_1) \quad (60)$$

$$\text{expansion work : } w_{\text{exp}} = u_3 - u_4 = C_P(T_3 - T_4) \quad (61)$$

$$\text{cycle net work : } w = w_{\text{exp}} - w_{\text{comp}} \quad (62)$$

Thermal efficiency,  $\eta_{th}$ , of a cycle is defined as the ratio between the cycle net work and the heat added, i.e.:

$$\eta_{th} = \frac{w}{q_H} \quad (63)$$

By substituting Eqs. 58, 59 and 62 along specific heats ratio, it yields;

$$\eta_{th} = 1 - \frac{q_L}{q_H} = 1 - \frac{C_V(T_4 - T_1)}{C_P(T_3 - T_2)} = 1 - \frac{T_4 - T_1}{\gamma(T_3 - T_2)} \quad (64)$$

As the process 1–2 is isentropic, then

$$T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{\gamma-1} = T_1 r_v^{\gamma-1} \quad (65)$$

Where,  $r_v$  is the compression ratio. In Diesel cycle fuel is injected into the combustion chamber up to a certain point known as the cutoff ratio defined by

$$r_C = \frac{V_3}{V_2} \quad (66)$$

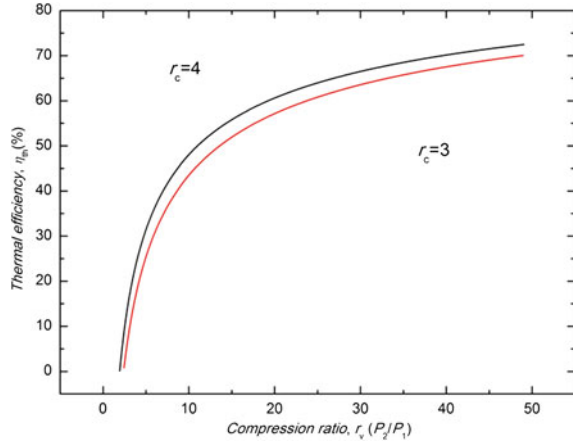
Also, after a few manipulations it is possible to relate  $T_4$  with  $T_1$ , which is

$$T_4 = T_1 r_C^\gamma \quad (67)$$

Also, considering the process 2–3 is an isobaric one, then

$$T_3 = T_2 r_C \quad (68)$$

**Fig. 14** Diesel cycle efficiency as a function of the compression ratio,  $r_v$ , and cutoff ratio,  $r_c$



By substituting Eqs. 65 through 68 into Eq. 64, one obtains

$$\eta_{th} = 1 - \frac{1}{r_v^{\gamma-1}} \left[ \frac{r_c^\gamma - 1}{\gamma(r_c - 1)} \right] \quad (69)$$

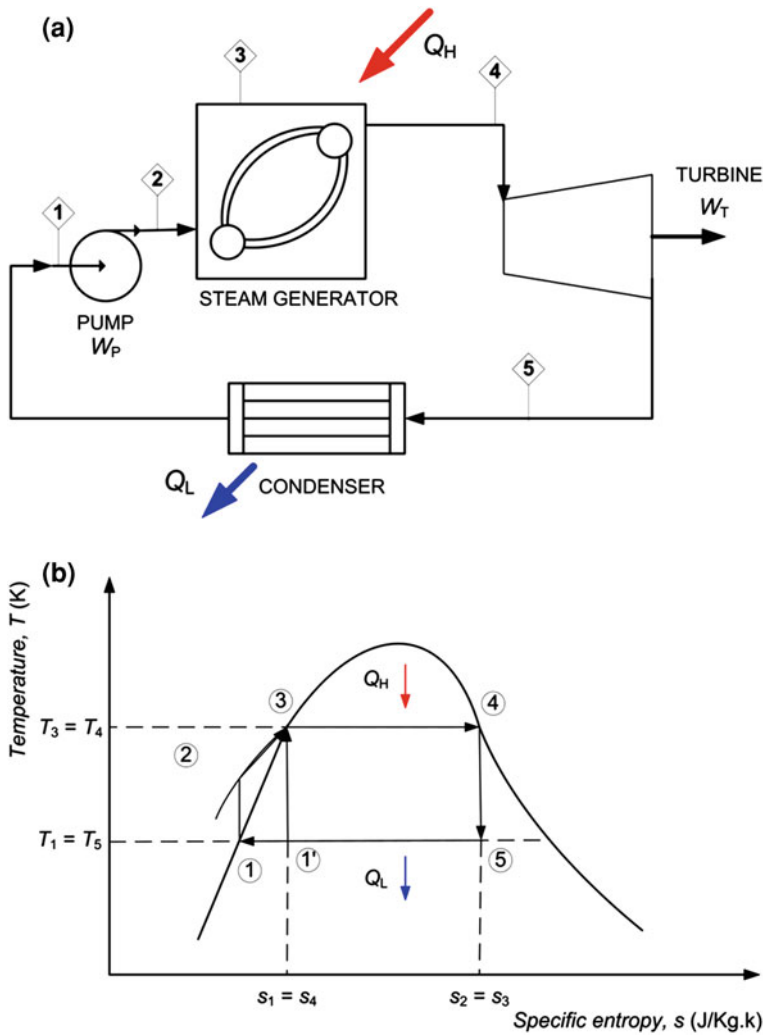
Figure 14 shows the Diesel cycle efficiency as a function of the compression ratio,  $r_v$ , and cutoff ratio,  $r_c$ .

## 4 Rankine Cycle

Rankine cycle is the one used in steam power plants. The most common fluid used in this cycle is water, but other fluids can also be used. Lately, ROC, Rankine Organic Cycles have been devised using organic fluids, rather than water. ROC is mostly used in small to medium installations and they are usually powered by solar energy or recovered waste heat. Industrial and large thermal power plants use conventional Rankine Cycles, which are revised in this section. First, the simplest Rankine cycle is presented and the necessary variations are discussed until discussing the more commercial configurations.

### 4.1 The Simple Rankine Cycle

The simplest Rankine cycle is the one based on four reversible process as shown in Fig. 15a. Saturated liquid 1 undergoes an isentropic compression process to reach compressed liquid at state 2. Next, the compressed liquid is driven to the steam generator, where heat  $Q_H$  is added to obtain saturated vapor at state 4. Useful work is produced in an expansion machine, such as a steam turbine, in an isentropic process



**Fig. 15** a Four basic components of a simple Rankine cycle; b temperature-specific entropy diagram and Carnot cycle

yielding fluid at state 5. Finally, there occurs condensation by removing heat  $Q_L$  in the condenser to close the cycle and the fluid returns to the initial state 1. All processes are ideal. The diagram  $T-s$  in Fig. 15b also shows the corresponding Carnot Cycle 1'-3-4-5-1'. Clearly, one can see that the Carnot cycle has a higher thermal efficiency than the simple Rankine cycle by simply reasoning that heat is delivered to the Rankine cycle at an average temperature (between  $T_2$  and  $T_H$ ) lower than the one for the Carnot cycle ( $T_H$ ).

Thermal balance around the pieces of equipment of the Rankine cycle are:

$$\text{heat addition (steam generator)} : q_H = h_4 - h_2 \quad (70)$$

$$\text{heat rejection (condenser)} : q_L = h_5 - h_1 \quad (71)$$

$$\text{compression work (pump)} : w_p = h_2 - h_1 \quad (72)$$

$$\text{expansion work (turbine)} : w_t = h_4 - h_5 \quad (73)$$

$$\text{cycle net work} : w = w_t - w_p = q_H - q_L \quad (74)$$

Magnitudes in Eqs. 70 through 74 are on mass basis. For instance, if one needs the cycle total net power,  $W$ , it may be obtained according to Eq. 75, i.e.,

$$W = \dot{m} \times w \quad (75)$$

where,  $\dot{m}$  is the mass flow rate. It is also a common practice to obtain the ideal pumping work by the following expression

$$w_p \cong v_1 \times (P_2 - P_1) \quad (76)$$

## 4.2 Rankine Cycle With Vapor Superheating

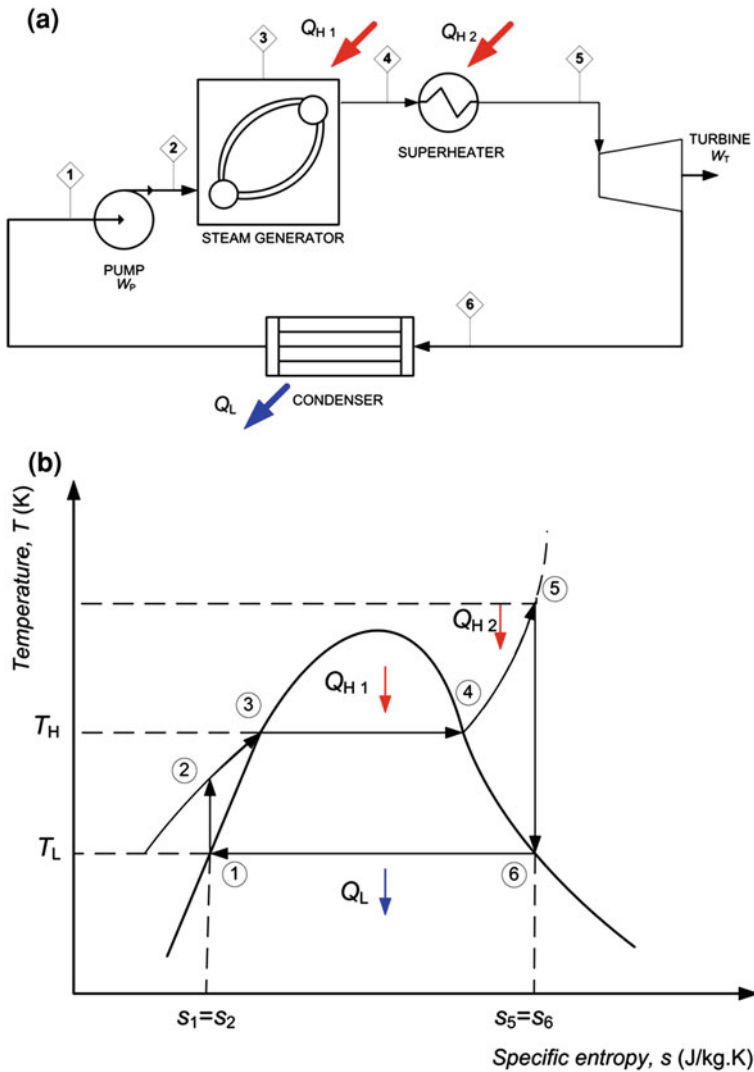
By closely examining the  $T$ - $s$  diagram of the simple Rankine cycle (Fig. 15b), it is possible to notice that at the exit of the expansion machine (turbine) a mixture of liquid and vapor is present (state 5). Usually, a vapor quality at and below around 90% can cause damage to the turbine blades by erosion due to the impact of droplets at high velocity on them. The way to get around the blade impact problem is done by introducing a first modification on the simple Rankine Cycle. Usually, a superheater is installed at the exit of the steam generator in order to superheat the saturated vapor to higher temperatures  $T_6$  as seen in Fig. 16a. Usually, the superheater is an additional piece of equipment integrated to the steam generator. The  $T$ - $s$  diagram is shown in Fig. 16b.

Clearly, by heating up the working fluid to higher temperatures, a higher thermal efficiency will also be obtained without any additional increase in the working pressure. However, there is an additional cost of the superheating stage installation.

## 4.3 Rankine Cycle With Vapor Reheating

The previous Rankine cycle configuration can solve the problem of wet steam at the turbine exit. However, it brings about a new problem that is to superheat the turbine inlet temperature to a considerable high value. To solve this, the solution is





**Fig. 16** a Rankine cycle with vapor superheating; b temperature-entropy diagram

to expand the vapor to a intermediate pressure and direct the vapor back to the steam generator to reheat it. Next the superheat vapor is expanded in a second stage of the steam turbine. The schematics of this configuration can be seen in Fig. 17a. The  $T-s$  diagram is shown in Fig. 17b. What really is done is to expand the vapor in stages so that the expansion process progresses around the vapor saturation curve in a way such vapor quality is not too high in the end of each stage. Figure 17a shows a two-stage steam turbine, but additional stages are also possible.

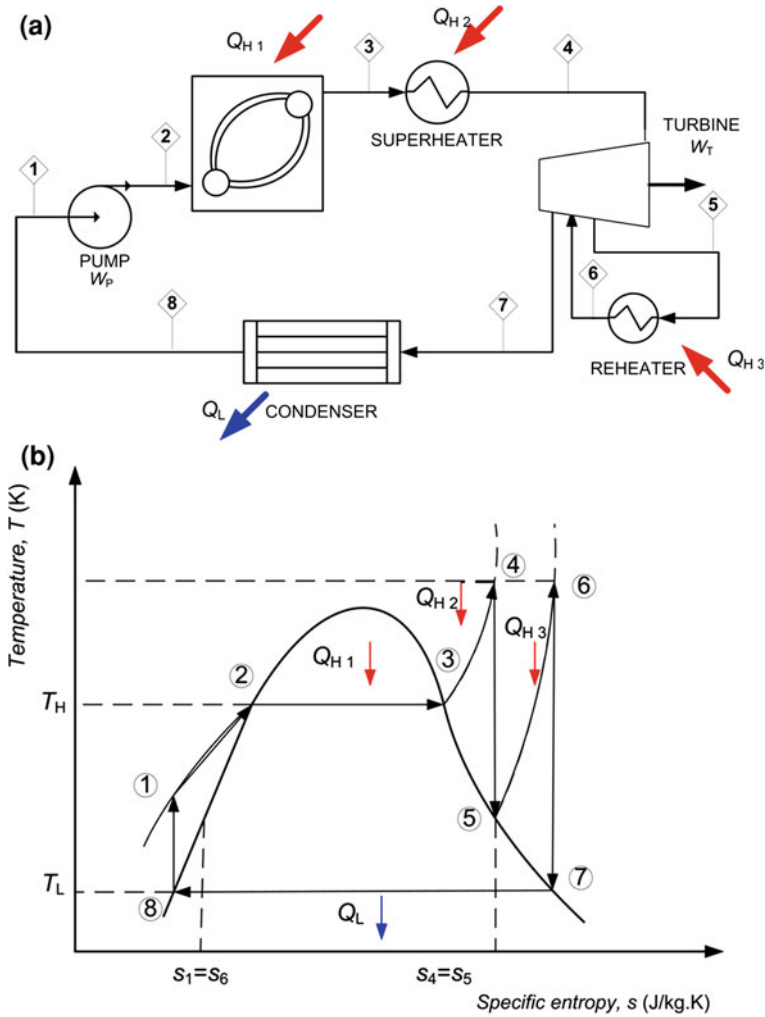
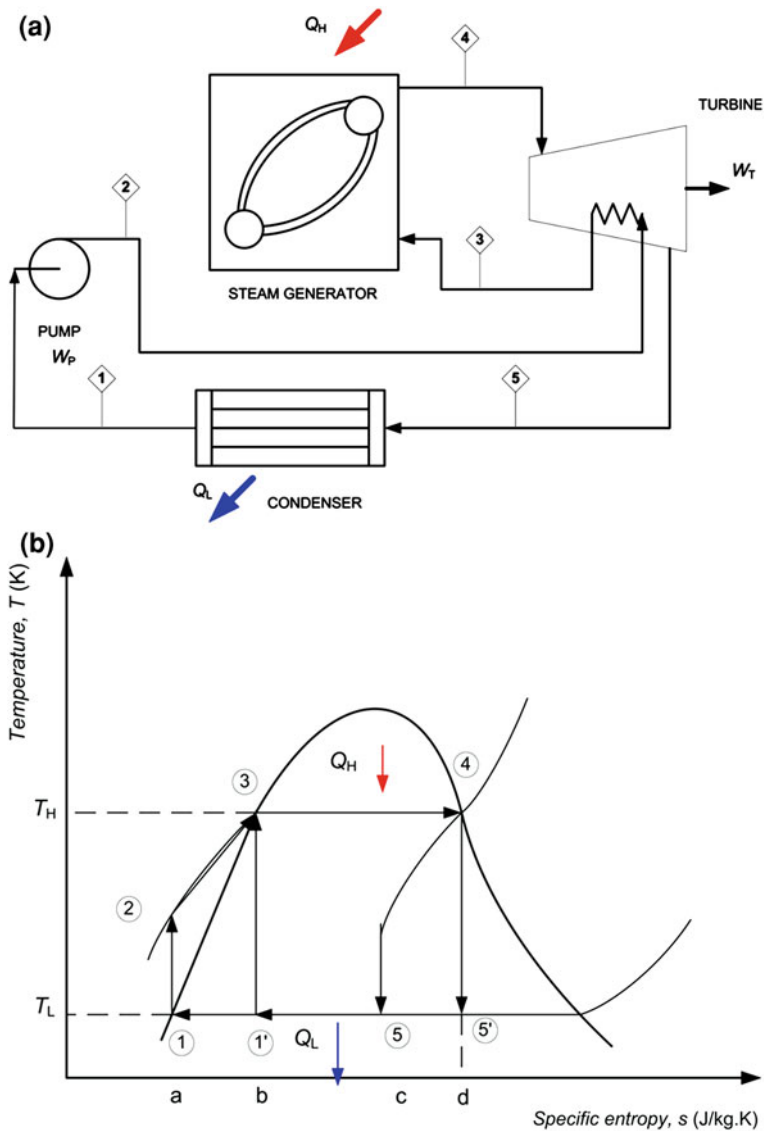


Fig. 17 a Rankine cycle with vapor reheating; b temperature-entropy diagram

#### 4.4 Regenerative Rankine Cycle

Neither one of the previous Rankine cycle variations solves the problem of lower thermal efficiency of the Rankine cycle when compared with the equivalent Carnot cycle. Part of the problem is related to the heat addition at a low temperature liquid that enters the steam generator coming from the pump. In order to solve this, a regenerative cycle was conceived.

In a regenerative cycle, the vapor does not expand isentropically in the turbine, but as it expands it exchanges heat with the compressed liquid that travels in a



**Fig. 18** a Regenerative Rankine cycle; b temperature-entropy diagram

counterflow configuration in the turbine frame. In doing so, the liquid is preheated and it will enter the steam generator as a saturated liquid. The schematic of an ideal regenerative cycle is shown in Fig. 18a. The  $T-s$  diagram is shown in Fig. 18b.

By examining the Fig. 18b, one can notice that ideally the regenerative cycle reproduces exactly the Carnot cycle because the exact amount of heat used to

pre-heat the compressed liquid is equal to the heat lost during the expansion (areas under the curves). However, the regenerative cycle is not a practical one for at least two main reasons. First, it is not practical to design a steam turbine that it is at the same time an expanding machine and a heat exchanger. Second, the vapor quality of the vapor exiting the steam turbine is considerably lower, which can enhance the blade erosion problem. The practical solution to is extract vapor from turbine stages and mixture it with condensation water as it is examined in next section.

#### 4.5 Regenerative Rankine Cycle With Feed Water Heating

The ideal regenerative cycle cannot be achieved in practice because of the mentioned problems. Usual practice consists of making partial expansion in the steam turbine, extract part of the vapor and mixes it with condensing water in a feedwater heater as indicated in Fig. 19a. The corresponding  $T$ - $s$  diagram is shown in Fig. 19b. The thermodynamic analysis is a little more difficult because the equations now must take into account that only part of the vapor continues the expansion ( $1-m_1$ ), while the other part ( $m_1$ ) will undergo mixing with the condensing water.

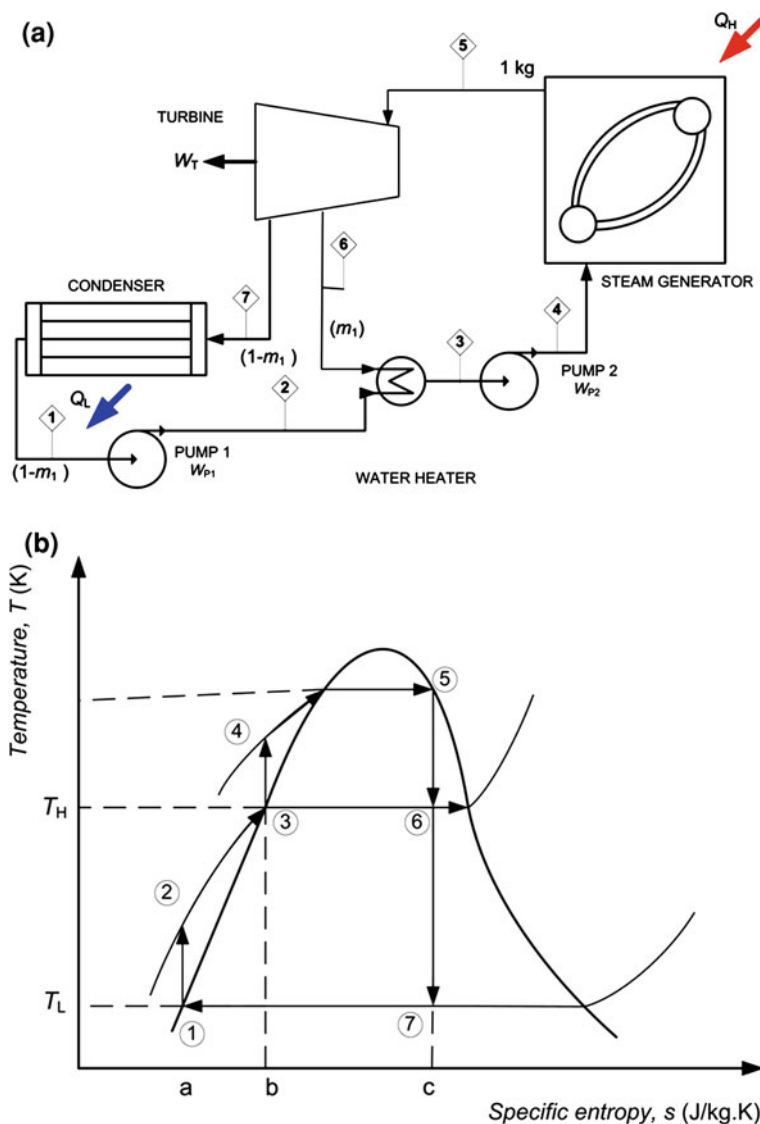
A more complex cycle with three feedwater heater is shown in Fig. 20a. The corresponding  $T$ - $s$  diagram is shown in Fig. 20b. An infinite number of feedwater heaters would reproduce the ideal Regenerative Rankine Cycle. This is evidently impossible In practice, six or seven heaters is usually the number of extractions and feedwater heaters used in large power plants. A detailed analysis of the optimum number of feedwater is given by Salisbury.

#### 4.6 Losses in Rankine Cycles and Alternative Working Fluids

The most common machine to produce net shaft work in Rankine Cycles are steam turbines. As an actual device, a steam turbine is not an isentropic one and internal losses due to fluid friction with turbine blades, aerodynamics losses as the vapor flows around the blades as well as entrance and exit losses deviate the expansion process from the ideal one. In order to compute losses altogether it is usual to define the turbine isentropic efficiency as,  $\eta_t$ , as it was also done for the gas turbine (Eq. 52). Therefore, the actual turbine work,  $w_{t-a}$ , is obtained from the ideal turbine work,  $w_{t-s}$ , as:

$$\eta_t = \frac{w_{t-a}}{w_{t-s}} = \frac{w_{t-a}}{h_i - h_{o-s}} \Rightarrow w_{t-a} = \eta_t(h_i - h_{o-s}) \quad (77)$$

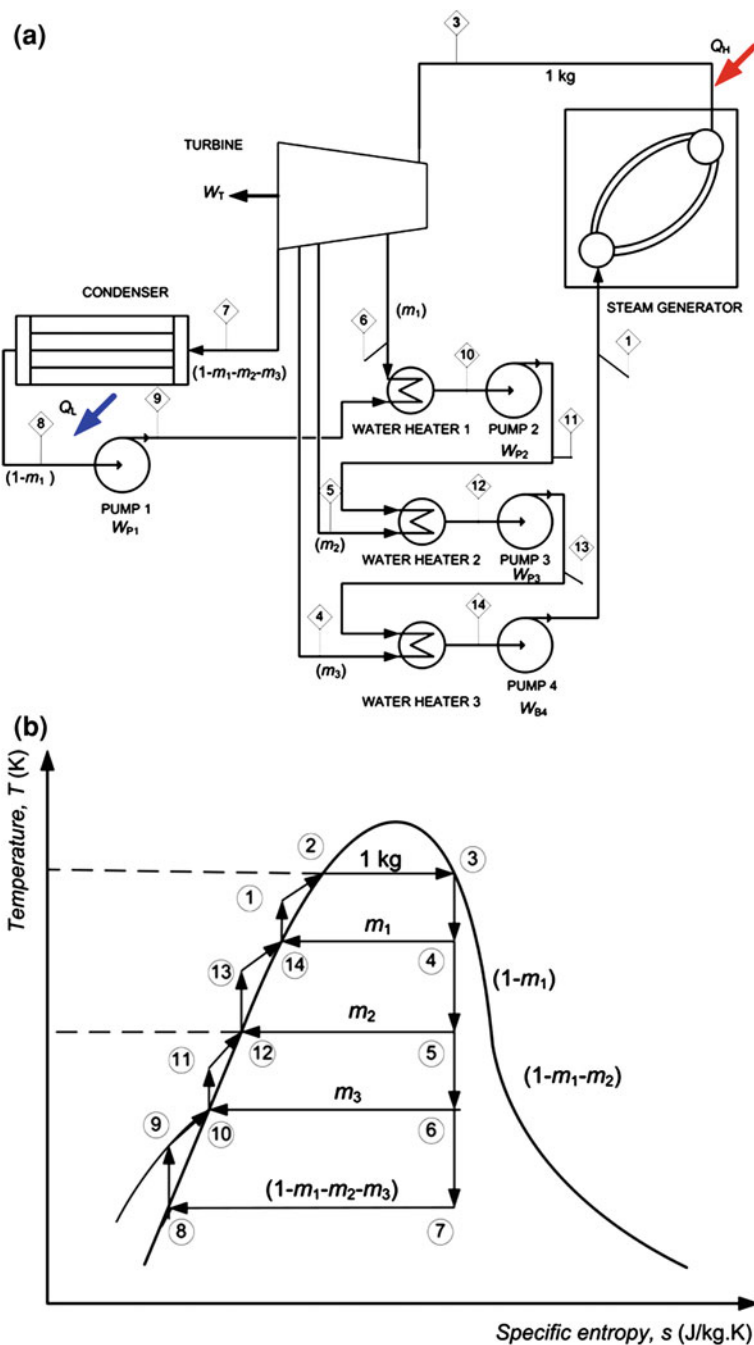
where,  $h_i$  is the specific enthalpy of the vapor at the turbine inlet an  $h_{o-s}$  is the specific enthalpy at the turbine exit in an isentropic process.



**Fig. 19** **a** Regenerative Rankine cycle with feed water heater; **b** temperature-entropy diagram

Pumps are also not ideal pieces of equipment and it is required to take that into account. Other losses are associated with working fluid flow in pipes and accessories which cause distributed and local pressure losses. Finally, heat addition and rejection are not isothermal.

Huang (1988) lists a series of requirements for an ideal working fluid to operate in Rankine cycles, which are partially reproduced below.



**Fig. 20** a Regenerative Rankine cycle with three three vapor extractions and three feed water heaters; b temperature-entropy diagram

- (1) It is desirable a high critical temperature. That would allow to work at high vaporization temperature;
- (2) High vaporization enthalpy. A high vaporization enthalpy reduces the amount of mass flow rate for the same heat power added to the cycle.
- (3) Condensing pressure positive. This is an important requirement, as the lower pressure in the system occurs in the condenser. External air can penetrate in the system.
- (4) Positive slope of the saturation vapor curve in the temperature-entropy diagram. A fluid with a saturation curve with positive slope would avoid the use of additional pieces of equipment such as superheater and reheaters. A proper study of this class of fluids known as retrogrades can be found in Thompson and Sullivan [2];
- (5) High density at operating temperatures and pressures. This would minimize the size of equipment;
- (6) Nontoxic, noncorrosive and chemically stable;
- (7) Low cost and widely available.

Water is the most used working fluid, but it fails to fulfill part of the previous requirements. On the positive side, water satisfies items 1 (partially), 2, 6, and 7. On the other hand, at condensing temperatures around 40°C the saturation pressure is 7.4 kPa and care must be taken to make the condenser air tight. Also, the accentuate (negative) slope of saturation vapor curve ( $T$ - $s$  diagram) demands the use of superheater and reheater to avoid a high degree of wet steam at the exit of the steam turbine. Nowadays, ROC, Rankine Organic Cycles have been used in the context of solar power plants that are considering the usage of other fluids rather than water.

## 5 Combined Brayton-Rankine Cycle

As analyzed in Sect. 2, a relatively high exhaust gas temperature is obtained in gas turbines. Considering economic and environmental issues it makes sense to recover such amount of thermal energy to produce other useful effect. One possibility is just to use the regeneration technique as described in Sect. 2.3. Other possibilities include; (1) using the exhaust gases to power an absorption refrigeration cycle. As a result air conditioning can be obtained for controlling the environment in working areas or, even, to cool off the gas turbine inlet air temperature, which can boost its power capacity (lower  $T_1$  in Eq. 47); (2) using a compact heat exchanger to heat a fluid for any process purpose, including hot water and steam; and (3) using a heat recovery steam generator (HRSG) to produce steam at a temperature and pressure enough to power a steam turbine (or other expansion machine) in a Rankine cycle. Only the latter case is studied here.

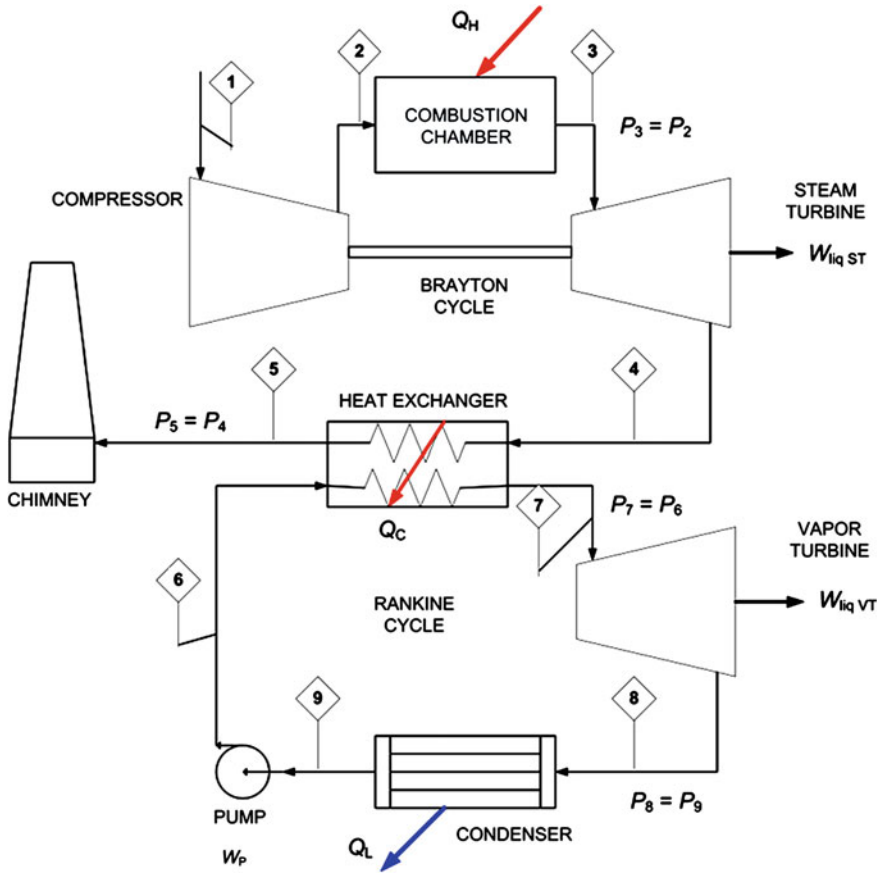


Fig. 21 Combined Brayton-Rankine cycle in 1 to 1 configuration

Figure 21 shows a schematic of a combined Brayton-Rankine cycle. Notice that steam to power the Rankine cycle is produced in the HRSG, which is driven by the exhaust gases from the Brayton cycle. As a consequence, the overall cycle efficiency is improved as seen next.

The combined cycle efficiency,  $\eta_C$ , is given by

$$\eta_C = \frac{W_{net}}{Q_H} = \frac{W_B + W_R}{Q_H} \quad (78)$$

where,  $W_B$  is the net shaft power produced by the gas turbine (Brayton cycle),  $W_R$  is the net shaft power produced by the Rankine cycle, and  $Q_H$  is the thermal power due to fuel combustion. The pumping power in the Rankine cycle has been neglected. Now, by substituting the thermal efficiency of the Rankine cycle,  $\eta_R$ , yields:



$$\eta_C = \frac{W_B + \eta_R \times Q_C}{Q_H} \quad (79)$$

where,  $Q_C$  is the heat load in the HRSG transferred from to exhaust gases to produce steam. Considering an ideal condition in which the exhaust gases leaving the HRSG are at a low temperature, then:

$$Q_C = Q_H - W_B \quad (80)$$

Substituting Eq. 80 into (79) along with cycles thermal efficiencies, yields:

$$\eta_C = \frac{W_B + \eta_R \times (Q_H - W_B)}{Q_H} = \eta_B + \eta_R - \eta_B \times \eta_R \quad (81)$$

Evidently, a combined cycle reaches higher thermal efficiencies than that of single cycle. For example. Consider a 40% thermal efficiency of a Brayton cycle and a 30% Rankine cycle thermal efficiency. The combined cycle efficiency is 58%.

The configuration in Fig. 21 is of the type 1–1, i.e., one gas turbine for one steam turbine. Other configurations are also possible depending on the individual machines. For a large gas turbine, one can think of a 1–2, i.e., one gas turbine and two steam turbines. Also it is possible 2–3, or two gas turbines or three steam turbines. It is a matter of matching machines capacities.

A combined Diesel-Rankine cycle is also possible. As a general rule, exhausting gases temperature of a Diesel engine is lower than the one from a gas turbine without a heat regenerator. Nevertheless, it is also possible to match working conditions for a lower pressure Rankine cycle.

As a final word on combined cycles, not only a simple thermal balance may result in operating cycles. The HRSG has some operational constraints depending on the temperatures and flow rates as well as the operating vapor pressure levels. It is not the goal of this chapter to discuss that problem, but approximation and pinch temperatures are issues to be analyzed on selecting power machines for operation in combined cycles.

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